The Question

Jed asked:

For integer $x > 0$, let $a_x > 1$ be the least integer such that $\left\lfloor \frac{x}{a_x} \right\rfloor = \left\lfloor \frac{x}{a_x - 1} \right\rfloor$.

Finding a Lower Bound for a_x

Dispending with minimality for a moment, a necessary condition on the value of a_x is:

$$\begin{aligned}
\lfloor \frac{x}{a_x} \rfloor &= \lfloor \frac{x}{a_x - 1} \rfloor \\
\implies \frac{x}{a_x - 1} - \frac{x}{a_x} &< 1 \\
\iff \frac{x}{a_x(a_x - 1)} &< 1 \\
\iff a_x(a_x - 1) &> x \\
\iff (m + 0.5)(m - 0.5) &> x \text{ (where } m = a_x - 0.5) \\
\iff m^2 - 0.25 &> x \\
\iff m^2 &> x + 0.25 \\
\iff m &> \sqrt{x + 0.25} \\
\iff a_x &> \sqrt{x + 0.25} + 0.5
\end{aligned}$$

So start your search here (or perhaps at some smaller value that's more efficient to calculate in your setting).